Chapter 2 Fundamentals of the Reliability-based Design Method

1.1 Summary of the Reliability-based Design Method

1.1.1 General

"Structure reliability" is defined broadly as "the ability for structures to satisfy the required design performance during a specific period," and, in a narrow sense, as "the probability that structures do not reach the limit state during their working life." On the other hand, since failure probability is "the probability that structures reach the limit state during their working life," the failure probability and reliability are related as "failure probability = 1.0 − reliability."[1] This leads to an understanding that reliability analysis is "a method to quantitatively describe the uncertainty of individual basic variables related to the performance of structures based on the probability distribution, and calculate the failure probability based on probability theory." The reliability-based design method can be called "a design method to determine how to design based on the result of reliability analysis."

"Technical Standards and Commentaries for Port and Harbour Facilities in Japan (2007)" adopted the reliability-based design method at a full scale. The reliability-based design method has been adopted worldwide by the Structural Eurocodes (a European uniform standard) and by the AASHTO (a leading design standard in North America), among others. It should be noted that international standards such as "ISO 2394: General principles on reliability for structures" also apply the reliability-based design method a standard design method,[2] and it is currently recognized as the only method that can reasonably treat various uncertainties unavoidable in structural design.

This book also describes the reliability-based design method as a representative performance verification method. This chapter describes general ideas and technical details[3] regarding the reliability-based design method in the context of this book.

1.1.2 Classification of the Reliability-based Design Method

The classification of the reliability-based design method[1), 4), 5)] is described in **Part I, Chapter 1, 3.9.1 (2)** and is restated here:

(1) Level 3 Reliability-based Design Method

A reliability-based design method for directly calculating failure probability using a probability calculation based on the probability distribution of each basic variable.

(2) Level 2 Reliability-based Design Method

A reliability-based design method for evaluating the reliability index β using a probability calculation based on the average, distribution, and covariance of each basic variable (not dependent on the probability distribution), and evaluating performance so that β is equal to or exceeds the limit value.

(3) Level 1 Reliability-based Design Method

A reliability-based design method for deterministically verifying performance by introducing some partial factors to the performance verification equation and substituting the equation with characteristic values for the basic variables in order to ensure a safety margin. This is also called the "partial factor method." The partial factor is determined with the level 3 or 2 reliability-based design method, which is called the "code calibration."

1.2 Basic Concept of Failure Probability and Target Safety Level

1.2.1 Notes on Reliability Analysis

With progress being made in research on the reliability-based design method, the classification and interpretation of various uncertainties concerning design and the quantification of such uncertainties (estimation of probability distribution) due to limited available data are being recognized as significant problems, especially when dealing with naturally-derived materials such as external forces and ground, as well as errors in modeling. Based on the above, the reliability analysis can be understood as "a method to methodically sort and systemize different factors regarding uncertainties related to the performance of structures, statistically quantify individual factors of uncertainties, and calculate the failure probability according to the probability theory."

On the other hand, organization of individual factors of uncertainties and their adequate statistical quantification may not always be possible in actual reliability analysis. Specific reasons are exemplified below:

(1) Difficulties in the Classification of Uncertainties

Literature on traditional structural reliability analysis classifies factors of uncertainties in structure design as below:[2)]

① Spatiotemporal variation of external forces acting on structures, mechanical properties of materials, and physical quantities such as geometric shape.

② Statistical estimated errors due to the estimation of uncertainties in physical quantities from samples.

③ Errors in modeling due to the simplification and idealization of design calculation models. Errors in modeling are generally defined by the following equation:

$$\text{Errors in modeling} = \frac{\text{Actual (observed) response}}{\text{Response expected by a model}} \tag{1.1.1}$$

④ Conversion errors when converting results from different surveys to physical quantities directly necessary for design.[6)–8)]

However, quantification of uncertainties as in the above classification may not necessarily be possible since actual analysis of uncertainties produces a variety of uncertainty-related data from individual structures. It may be difficult to individually extract these uncertainties in many cases and often only multiple uncertainties can be estimated.

For example, a traditional design method may be able to estimate uncertainties in the whole design method using a full scale experiment or examples of damage, even if it is unknown how the factors of uncertainties in individual basic variables contribute. In some cases, uncertainties may also be suspected to have been double-counted or missed. Proper engineering judgement, including sorting data, by applying comprehensive knowledge of structure design and considering details regarding the acquired data is needed in specific aspects of each reliability analysis.

(2) Difficulties in the Quantification of Uncertainties

The quantification of uncertainties always requires estimation using some statistical data. However, these data always have inherent qualitative and quantitative problems. Restriction in the number of data and uneven distribution of many collected data ranges require close attention to statistical analysis for the quantification of uncertainties, while there are unavoidable limitations. In some cases, certain factors make it impossible to quantify uncertainties based on data, which forces quantification based on an experienced engineer’s expert judgement.

1.2.2 The Concept of Failure Probability as an Absolute Criterion of Structural Reliability in This Book

The reliability analysis has many problems as mentioned above in stages prior to probability calculation based on the probability theory, and the degree also differs by the target problem. Thus, it may be doubtful if uncertainty, or the failure probability, built upon certain assumptions is an absolute criterion of structural reliability. This is the reason why this book considers that comparisons of failure probabilities between different kinds of structures and making judgements by overestimating failure probability as an absolute criterion should be withheld at this time.

On the other hand, it was understood that the relative comparison of reliability analyzed by similar methods based on homogeneous data was effective, so a performance verification method was constructed in this book. This implies that making comparisons of failure probabilities of structures of the same type is effective to a considerable degree.

1.2.3 Main Methods to Determine a Target Safety Level

A target safety level is mainly determined using the following three methods:

① Setting a similar reliability level of a structure designed by existing design standards as the target safety level.

② Setting the target safety level by comparing it to different background risks.

③ Setting by optimizing economic indices such as life cycle costs and expected costs.

(1) Currently, method ① is often used because structures designed using existing design standards usually already have long performance records, and thus, the reliability of such structures is considered to have been socially recognized. Moreover, the fact that there is no sufficient confirmation if the value of reliability obtained by current reliability analysis is meaningful in every case, as an absolute criterion of reliability is considered to be another reason why many calibrations are done using this method. By contrast, a relative comparison of the reliability of homogeneous

structures is considered to be sufficiently meaningful. The level of reliability of homogeneous structures is generally equalized with the code calibration in this method.

(2) Method ② using background risk is utilized for different policymaking based on risks in many areas. Proper setting of environmental limit values of chemical substances is the typical example.[9] This method intends to determine the target safety level of structures by referring to various risks of events existing in society. Examples of degrees of risks existing in society are summarized in Honjo et al.[10]

(3) Method ③, which intends to minimize so-called expected total costs (construction cost plus expected failure cost, which is obtained by multiplying damage, including social costs when the structure fails, by the failure probability), appears to be reasonable.[11] However, as there are many problems such as accuracy of failure probability as an absolute criterion, cost estimation when failed, and forecasts of socioeconomic changes within the working lives of structures, this method seems to pose many problems for actually calculating the target safety level at this time.

(4) The target safety level may be set after evaluating it as the system reliability for multiple limit states of structures. This method, for example, evaluates system reliability as the reliability of a serial system[12] after calculating the failure probability using the failure mode of sliding, over-turning, and bearing capacity at a gravity-type wharf. In this case, the whole system is considered to fail if either of the above failure modes occurs.

However, the impact the limit state of each failure mode gives to the performance (stability) of the whole structure can differ by failure mode, and there is a possibility that the conventional design method (safety factor method) has also taken this into account when considering a safety margin. Consequently, this book considers that reasonable partial factors can be set by conducting calibration targeting reliability of current design methods for each limit state rather than mechanically calculating the system reliability. This book, therefore, does not take account of system reliability when setting the target safety level.

1.2.4 Setting Processes for the Target Safety Level in This Book

The "Technical Standards and Commentaries for Port and Harbour Facilities in Japan (2007)" fully introduced the reliability-based design method as a performance verification method. As stated above, classification and organization of factors of uncertainties as a basis are important in reliability analysis. When presenting a verification equation using partial factors regarding the structural form and the failure mode on which the reliability was analyzed in this book, close investigation of factors of uncertainties included in the failure mode and so on, which are conceivable at this time in reliability analysis, led to a review of everything including the target safety level in the reliability analysis. As for the reasoning behind these settings, see the references listed in the "Facilities" section. **Fig. 1.2.1** below shows a basic flow up to the setting of partial factors.

1. Overview and review of the safety level
- Comparison between criteria (the 1999 Criteria and the 2007 Criteria)
- Review of target safety levels (as required)

▼

2. Reliability analysis utilizing the Monte Carlo simulation
- Calculation of the failure probability
- Setting of a target failure probability (by failure mode)

▼

3. Setting of partial factors in the load and resistance factor design
- Comparison and examination of partial factor formats
- Evaluate conformance to target failure probability and effect to design result (including convenience), and adopt a partial factor format which requires as few partial factors as possible.

Fig. 1.2.1 Basic Flow up to the Setting of Partial Factors Applied in This Book

1.3 Adoption of a Load and Resistance Factor Design for the Level 1 Reliability-based Design Method

1.3.1 General

The form of performance verification equations adopted in this book (a.k.a. "format") is a partial factor method based on a load and resistance factor design (hereinafter called the "load and resistance factor design"), which is different than the partial factor method based on a material factor design adopted in the "Technical Standards and Commentaries for Port and Harbour Facilities in Japan (2007)" (hereinafter called the "material factor design"). This section describes the differences between these formats and concepts, and the reasons why the load and resistance factor design was finally adopted.

1.3.2 Partial Factor Method

Here, the material factor design and the load and resistance factor design are defined as follows, respectively. The term "partial factor method" collectively indicates both approaches and may be understood as being synonymous with the level 1 reliability-based design method. For details, please refer to Honjo et al.[10), 13)]

(1) Partial Factor Method Using a Material Factor Design

A material factor design is a performance verification method constructed with the concept of processing different uncertainties in performance verification at their sources. It features the introduction of many partial factors and processing of uncertainties in proximity to their sources. For example, it evaluates a design value by multiplying a characteristic value of each material by a partial factor, then substitutes the design value into the performance verification equation to verify the performance.

$$R\left(\gamma_{r1}x_{rk1},\dots,\gamma_{rn}x_{rkn},\gamma_{R1'},\dots,\gamma_{Rn'}\right) \geq S\left(\gamma_{s1}x_{sk1},\dots,\gamma_{sm}x_{skm},\gamma_{S1'},\dots,\gamma_{Sm'}\right) \quad \textbf{(1.3.1)}$$

where

γ_{ri} : partial factor to multiply characteristic value x_{rki} (a basic variable on the resistance side)

γ_{si} : partial factor to multiply characteristic value x_{ski} (a basic variable on the load side)

These partial factors are applied considering physical uncertainties inherent to materials and loads and statistically estimated errors. $\gamma_{Ri'}$ and $\gamma_{Si'}$ are partial factors applied to the modeling errors, redundancy, and vulnerability in design calculations. Partial factors are adjusted to ensure a safety margin finally necessary for structures. Note that the material- or load-related basic variables substituted into the performance verification equation in the material factor design are not characteristic values before being multiplied by a partial factor, but are values already multiplied by a partial factor (called "design values").

(2) Partial Factor Method Using a Load and Resistance Factor Design

A load and resistance factor design (LRFD) is a design concept that ensures a safety margin during performance verification by applying a partial factor to multiply a resistance value calculated based on a characteristic value. The load side introduces a partial factor to multiply the corresponding load value and increases the load. The LRFD often integrates resistance values, but a format to divide into several terms similar to the load side is also proposed. Here, these two forms are described together:

$$\gamma_R R\left(x_{k1},\dots,x_{kn}\right) \geq \sum_j \gamma_{Sj} S_j\left(x_{k1},\dots,x_{kn}\right) \quad \textbf{(1.3.2)}$$

$$f_R\left(\gamma_{R_1}R_{1k}(x_{1k},\cdots x_{pk}),\cdots,\gamma_{R_m}R_{mk}(x_{1k},\cdots x_{pk})\right) \geq f_S\left(\gamma_{S_1}S_{1k}(x_{1k},\cdots x_{pk}),\cdots,\gamma_{S_n}S_{nk}(x_{1k},\cdots x_{pk})\right) \quad \textbf{(1.3.3)}$$

where

γ_R or γ_{Ri} : partial factor to multiply a resistance value (hereinafter "resistance factor")

γ_{Si} : partial factor to multiply a load value (hereinafter "load factor")

R or R_i : resistance value

S_i : load (effect) value

In this way, the LRFD is a performance verification method that ensures a safety margin by calculating the resistance side as functions (simple addition in many cases) of (characteristic values of) several resistance values, multiplying each term by the resistance factor, and combining load factors.

The format of performance verification equations of **equations (1.3.2)** and **(1.3.3)** shown here looks slightly different from the equation shown in the "Common" section, but they are inherently the same. In other words, this book indicates that performance verification using the partial factor method may usually be conducted with the following **equations (1.3.4)** to **(1.3.6)**:

$$m \times \left(\gamma_i \frac{S_d}{R_d} \right) \leq 1.0 \quad \textbf{(1.3.4)}$$

$$S_d = f\left(\gamma_{S_1} S_{1k}, \cdots, \gamma_{S_n} S_{nk}\right) = f\left(\gamma_{S_1} S_{1k}(x_{1k}, \cdots x_{pk}), \cdots, \gamma_{S_n} S_{nk}(x_{1k}, \cdots x_{pk})\right) \quad \textbf{(1.3.5)}$$

$$R_d = f\left(\gamma_{R_1} R_{1k}, \cdots, \gamma_{R_m} R_{mk}\right) = f\left(\gamma_{R_1} R_{1k}(x_{1k}, \cdots x_{pk}), \cdots, \gamma_{R_m} R_{mk}(x_{1k}, \cdots x_{pk})\right) \quad \textbf{(1.3.6)}$$

where

S_d : design value of a response value

R_d : design value of a limit value

γ_i : partial factor to consider the significance of structures, social influence when the limit state is reached, etc. (structural factors). Unless otherwise specified, $\gamma_i = 1.0$ and is not described in this book.

m : adjustment factor (a value corresponding to the allowable safety factor in the traditional safety factor method or allowable stress method/working stress design. Equivalent to what was processed by the structural analysis factor in the 2007 version.)

S_{jk} : characteristic value of an effect of action j ($j = 1..n$)

γ_{Sj} : partial factor to multiply a characteristic value S_{jk} of an effect of action j

S_i ( ) : equation to calculate a characteristic value S_{jk} of an effect of action j

R_{jk} : characteristic value of resistance (strength) j ($j = 1..m$)

γ_{Rj} : partial factor to multiply a characteristic value R_{jk} of resistance (strength) j

R_j ( ) : equation to calculate a characteristic value R_{jk} of resistance (strength) j

x_{jk} : characteristic value of a basic variable x_j (j = 1..p)

If a partial factor is determined by calibration based on a reliability analysis, adjustment factor $m = 1.0$ and usually $\gamma_i = 1.0$. In this case, **equation (1.3.4)** coincides with **equation (1.3.3)**.

1.3.3 Merits and Demerits of a Partial Factor Method Using the Material Factor Design

The merits of a material factor design over a load and resistance factor design are as follows:

① Processing uncertainties at their source is intuitively reasonable.

② Because each uncertainty is processed at its source, the performance verification equation can be adapted to different situations by aligning the partial factor corresponding to the item where individual uncertainty changes. Practically, when a new construction method or technology is adopted, items concerning their reliability can be rapidly adjusted by aligning factors.

However, current knowledge hardly makes it possible to explain uncertainties in design as accumulated factors of individual uncertainties in the design of many structures. Adjusting uncertainties at their source could be unreasonable because:

① Not all uncertainties have been clarified individually and quantitatively.

② Uncertainties for the whole design method could be quantitatively understood if a load test or a failure example exists. However, it would be extremely difficult to divide the entirety of the uncertainties into individual factors and quantify them.

③ If a structure's behavior is forecasted with a design value adjusting a characteristic value by a partial factor, behavior deviating from the most plausible behavior of structures will increase. In designing structures (especially ground structures), it is recognized that the designer's engineering judgement is important, and they should design in principle by tracing the most plausible behavior of structures as much as possible up to the final design stage so that such judgements can soundly function. However, the material factor design is opposed to this principle.

1.3.4 Merits and Demerits of the Partial Factor Method Using a Load and Resistance Factor Design

The merits of a material factor design are demerits of the load and resistance factor design, and vice versa.

① Designers can trace the most plausible behavior of structures up to the final design stage using a load and resistance factor design which calculates design based on the characteristic values of basic variables. This conforms to the principle that designers should trace the most plausible behavior of structures as much as possible up to the final design stage so that their engineering judgement can soundly function.

② When designing ground structures which intricately interact with the ground, a reduction of basic variables, such as in the material factor design, does not necessarily act on the side of safety for the structural design.

③ If uncertainties regarding the design calculation model are obvious from examples of failures with full-scale structures, or from a database of test results in conditions close to the behavior of full-scale structures, the uncertainties are those which, as a whole, reveal many intervening factors of uncertainties. Uncertainties found in the behavior of existing structures can be realistically sought by introducing uncertainties obtained from these data. What can be determined based on these data is the resistance factor applied to calculated resistance values.

④ The load and resistance factor design is closer to the verification form of the safety factor method or allowable stress method/working stress design, which practitioners have become more accustomed to than the material factor design, and which feels more familiar once introduced.

1.3.5 Reasons for the Adoption of a Partial Factor Method Using a Load and Resistance Factor Design

The partial factor method using the load and resistance factor design has been adopted as a format of partial factors in this book because the merits for the load and resistance factor design are currently considered to be better than those for the material factor design as a format of the partial factor method in the performance verification equation for structures after organizing the pros and cons of both designs.

In particular, this is because the load and resistance factor design is a performance verification method with which a designer can trace the most plausible behavior of a structure as much as possible up to the final design stage. Furthermore, individual uncertainties in design are not necessarily obvious at this time, so uncertainties as a whole design method should be clarified by future data acquisition.

Note that the level 1 reliability-based design method is a convenient way to take account of the "economy of design" of standard structures (for efficient design). It is also possible to use the level 3 reliability-based design method directly for especially important structures.

1.4 Theory of Reliability Analysis

1.4.1 Failure Probability and Reliability Index

This section provides a simple explanation of the probability theory and terms necessary to understand the reliability analysis.

Fig. 1.4.1 shows a plane consisting of two random variables (action S and resistance R) and the condition of probability distribution on this plane. The function showing the probability distribution of the two variables is called the "simultaneous probability density function" and is represented by $f_{RS}(r, s)$. The probability theory defines the sum of probabilities that all events occur as 1.0, so the volume of the bell-shaped simultaneous probability density function shown in **Fig. 1.4.1** is 1.0.

The reliability analysis uses a performance function $g(R, S)$ which divides the S-R plane into a non-destruction range and destruction range, and defines the limit state plane as a plane when the function is 0. The non-destruction range and the destruction range correspond to positive values of $g(R, S)$ and negative values of $g(R, S)$, respectively. $g(R, S)$ is denoted Z ($Z = g(R, S)$). In this case, the failure probability calculates the volume of the simultaneous probability density function of the hatched area in **Fig. 1.4.1**. Here, we explained it using a reliability analysis problem composed of two variables that can be drawn, but the same explanation essentially applies to cases where the probability space and the performance function composed of multiple variables are introduced.

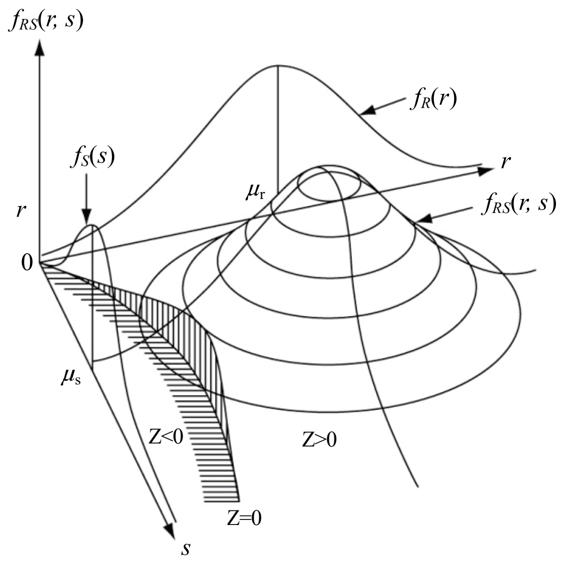

Fig. 1.4.1 Conceptual Diagram of Failure Probability Calculation

Fig. 1.4.2 shows the same concept as in **Fig. 1.4.1** with a slightly different representation. **Fig. 1.4.2** represents the bell-shaped simultaneous probability density function with contours and simplifies the performance function $Z = g(R, S)$ to a linear function R-S. The limit state plane separating the non-destruction range from the destruction range forms a positive 45-degree line from the origin. Because $Z = R$-S, a probability density function of the safety margin Z, shown in the figure, can be obtained when showing the volume of the simultaneous probability density function at $Z = z$ on the Z axis in a direction perpendicular to the limit state plane as shown in **Fig. 1.4.2**. The failure probability is also the area where Z is not positive in the probability density function.

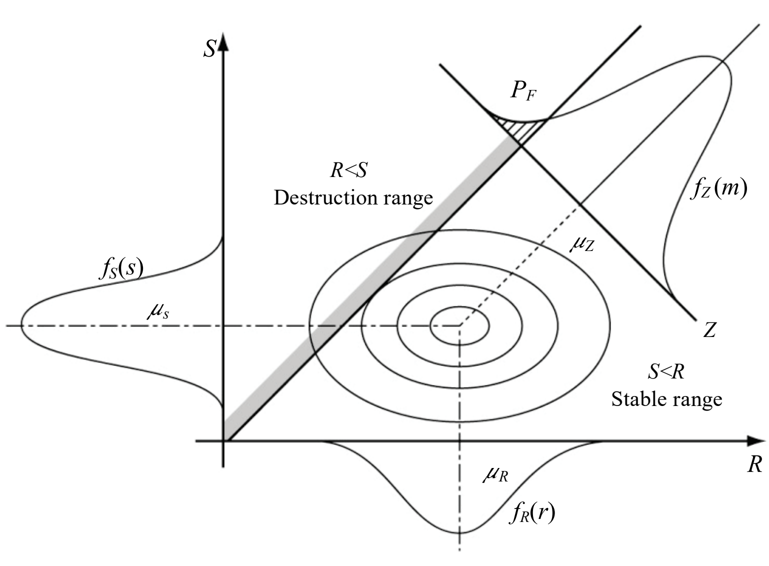

Fig. 1.4.2 Probability Density Function of the Safety Margin When the Performance Function Z = R-S

The average and variance of the safety margin Z are $\mu_Z = \mu_R - \mu_S$ and $\sigma_Z^2 = \sigma_R^2 + \sigma_S^2$, respectively, where the average of R is μ_R, the standard deviation is σ_R, the average of S is μ_S, and the standard deviation is σ_S. **Fig. 1.4.3** shows the probability density function of the safety margin Z in a different way. A reliability index β may be used instead of the failure probability P_f in the reliability-based design method. This reliability index is defined as follows using the average and variance of the safety margin:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad \text{or} \qquad \mu_Z = \beta\sigma_Z \tag{1.4.1}$$

The reliability index β is a scale to show the magnitude of margins that the average of the safety margin has over its standard deviation. Moreover, β can be directly connected to the failure probability P_f and is given in the following equation if the safety margin Z is normally distributed.

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{1.4.2}$$

Here, Φ is a standard normal probability distribution function and β and P_f have the relationship shown in **Fig. 1.4.4** and **Table 1.4.1**.

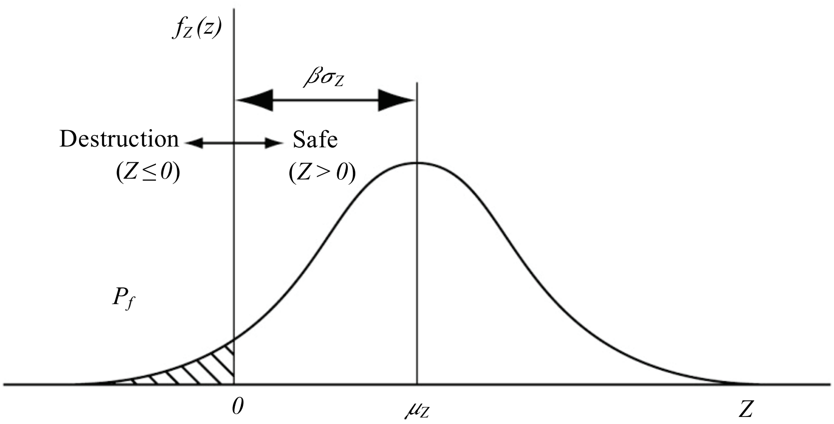

Fig. 1.4.3 Distribution of the Safety Margin Z and the Reliability Index β

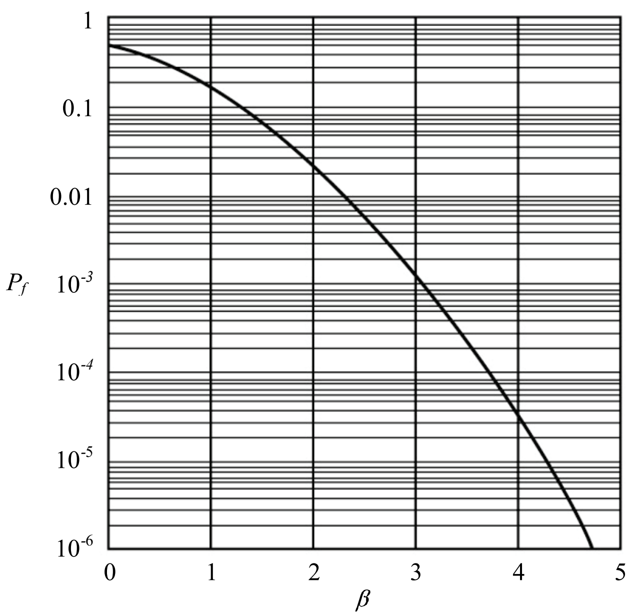

Fig. 1.4.4 Relationship between the Reliability Index β and the Failure Probability P_f

Table 1.4.1 Relationship between the Reliability Index β and the Failure Probability P_f

P_f	10^{-1}	5×10^{-2}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}
β	1.28	1.64	2.32	3.09	3.72	4.26	4.75

1.4.2 Calculation Method of Partial Factors with the Design Value Method and Notes

The code calibration to determine an actual partial factor value must take account of the following points irrespective of the format relating to the partial factor.

① There are many non-unique combinations of factors to secure structure reliability.

② It is difficult to find a combination of factors that gives neither too much nor too little reliability under any conditions since even structures of the same type are designed under different conditions.

① means that many combinations of partial factor values are possible because some partial factors are going to secure a safety margin which only one factor (safety factor) secured in the past.

② means that, for example, it is very difficult to secure uniform reliability with a combination of partial factors having the same value for quay walls of different water depths when considering reliability of gravity-type wharves.

The basic concept of the design value method is a proposed method to resolve the problem ① above and determine a partial factor value used in the level 1 reliability-based design method.[1), 2)] The basic concept of the design value method determines factors concerning all load values and resistance values, or all basic variables in relation to the design points and characteristic values. Design points in this context indicate points having the maximum likelihood (the maximum product of probability densities of basic variables) among points on the limit state plane (a plane the performance function of which is zero) in a space composed of load values and resistance values. **Fig. 1.4.5** shows design points ($S(x^*)$, $R(x^*)$) on the plane composed of load values $S(x)$ and resistance values $R(x)$. Therefore, it may be said that the design point is the point most plausible to reach the limit state among combinations of load values and resistance values on the limit state plane.

The design value method determines the load factor γ_S and the resistance factor γ_R using the following equation after determining the design point for structures having the target reliability as a reference point.

$$\gamma_S = S(\mathbf{x}^*) / S(\mathbf{x}_k), \qquad \gamma_R = R(\mathbf{x}^*) / R(\mathbf{x}_k) \tag{1.4.3}$$

where x_k indicates the characteristic value vector of the basic variable.

The design value method generally takes into account many cases that adequately cover the range of load specifications and ground conditions to which the design method is considered to be applied in code calibration. Then, load factors and resistance factors are evaluated by the design value method, and a partial factor which satisfies the given reliability for most of them is determined by soundly utilizing engineering judgement.

The design value method can also be considered a practical method to resolve the problem ① above. Therefore, it does not resolve the problem ② above, and in some cases, it may be a cross section of a level higher than the specified target safety level when calibrating codes concerning partial factors in this book, although the applicable conditions are sufficiently taken into consideration. As a result, note that there is an option in which a designer evaluates reliability by directly calculating the failure probability using the level 3 reliability-based design method as appropriate.

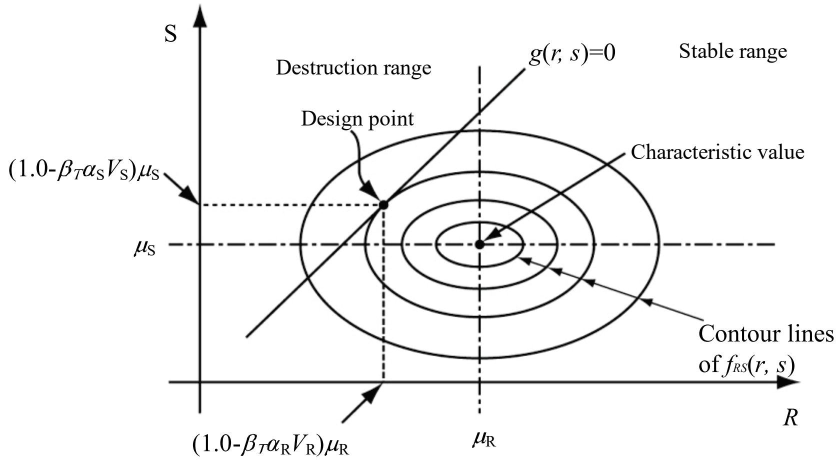

Fig. 1.4.5 Conceptual Diagram of the Design Points

1.4.3 Simple Calculation Theory of Partial Factors

The basic concept of the design value method has been explained above. Application of this method as it now requires repeated design calculations to determine design specifications that satisfy the target reliability under each design condition. In contrast to this, a method is proposed to determine an approximate partial factor without repeated design calculations. In particular, the following equation gives a load factor and resistance factor in the simplest case where the performance function is $Z = R\text{-}S$.

$$\gamma_R = \frac{\mu_R}{R_k}\left(1.0 - \beta_T \alpha_R V_R\right) \tag{1.4.4}$$

$$\gamma_S = \frac{\mu_S}{S_k}\left(1.0 - \beta_T \alpha_S V_S\right) \tag{1.4.5}$$

where

μ_R : average value of R

σ_R : standard deviation of R

V_R : variation factor of R

μ_S : average value of S

σ_S : standard deviation of S

V_S : variation factor of S

R_k and S_k are named for each characteristic value, β_T is the target safety index, and α_R and α_S are sensitivity factors, and are defined as follows:

$$\alpha_R = \frac{\sigma_R}{\sqrt{\sigma_R^2 + \sigma_S^2}}, \quad \alpha_S = -\frac{\sigma_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{1.4.6}$$

Equations (1.4.4) and **(1.4.5)** easily give a partial factor for any target safety index without finding a cross section which has the target safety reliability by repeated design calculations. However, note that this is a relationship that assumes that the sensitivity factor does not vary even when the target safety level varies to a certain extent. Moreover, instead of using this technique, this book calculates partial factors using the Monte Carlo simulation shown below.

All basic variables in the design value method are linear and are exact solutions when normally distributed, but are otherwise approximations. Details regarding these solutions are described in general textbooks on structural reliability, etc.[1)–5)]

1.5 Monte Carlo Simulation (MCS) as a Tool for Reliability Analysis

1.5.1 Advantages of Using the MCS

Structural engineers, among others, began a full-scale study of reliability analysis and reliability-based design method in the 1960s. Computers were still in their infancy, and streamlining the probability calculation through complex structural calculations was a major challenge. However, advances in computers and enormous progress made in random number generation methods made it possible to calculate probabilities using the Monte Carlo simulation (MCS). The MCS also has an advantage in that the calculation itself is intuitive.

Reliability analysis in this book was conducted using the MCS according to the following general procedure:

① Generate basic variable values (random numbers) constituting the performance function according to the probability distribution to which their variables conform.

② Calculate the performance function Z using the acquired combination of random numbers to evaluate if structures exceed the limit states concerned.

③ Conduct the above evaluation many times and obtain a failure probability by dividing the number of $Z \leq 0$ evaluations by the number of all trials.

As seen from this procedure, a reliability analysis using the MCS has many advantages such as easily understandable failure probability calculations, executable only with basic knowledge of probability calculation, and relatively hassle-free programming for probability calculation, and thus easily avoidable miscalculations of probability since it calculates deterministic performance functions multiple times.

Fig. 1.5.1 shows a result of the MCS analysis of stability against sliding of permanent state, gravity-type wharves.[14)] Although this MCS takes into consideration seven basic variables, the figure plots the total value (load value) of the calculated action side and the total value (resistance value) of the resistance side. The total number of trials is 500,000 and the estimated failure probability is 1.10×10^{-1} (=54,921/500,000).

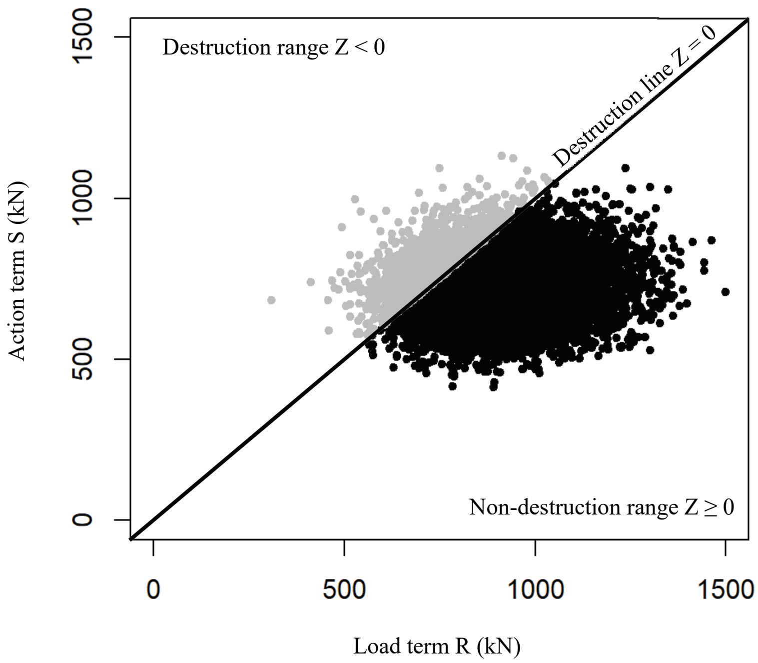

Fig. 1.5.1 Example of MCS Results of Sliding at Gravity-type Wharves[14)]

1.5.2 Setting of Partial Factors by the MCS

Calculation of partial factors using the MCS based on the design value method is relatively easy. Specifically, search for a design point with the highest likelihood among the points generated by the MCS and sufficiently close to the limit state, and calculate a partial factor with **equations (1.4.4)** and **(1.4.5)** using a characteristic value.

Refer to **Reference 14)**, which describes in detail an example of the calibration of load factors and resistance factors using the concept of design value method using the MCS found in this book.

1.5.3 Trial Times of the MCS

Significant progress in computing speed is one of the reasons why the MCS is frequently used for probability calculation. Dramatic progress in the generation method of uniform random numbers, which is a source for the generation of random numbers conforming to different probability distributions, also contributed to the use of the MCS. For example, Mersenne Twister, a current standard feature in computation languages such as R, MATLAB, and C++, and developed by Makoto Matsumoto and Takuji Nishimura (1998), is characterized by a very long period (2^{19937}-1) and high speed generation, and provides a highly reliable random number generation environment for the MCS in reliability analysis. Many standard computation languages functionalize random number generation according to different kinds of probability distributions and users can effortlessly obtain useful and highly reliable random numbers.

The number of MCS trials required for executing stable MCS is an important problem. One example of classical MCS literature, Rubinstein (1981),[15)] deals with this problem using basic probability theory. These results suggest that ensuring 50 to more than 100 samples in the destruction range is considered to be a guide for MCS trials necessary to evaluate stable failure probability. In other words, evaluation of a failure probability around 10^{-3} needs no less than approximately 10^5 MCS trials, and around 10^{-4} needs no less than approximately 10^6 MCS trials. It is also important to confirm that the failure probability converges as the number of trials increases.

Focused sampling is a typical method among several MCS methods proposed to efficiently estimate low failure probability. However, different characteristics of such methods require sufficient consideration of the nature of the problems before application. Low failure probability also needs to fully take account of the engineered significance of computed probability.

1.5.4 Other Supplementary Notes

Some basic variables comprising a performance function may be dependent and correlated. For example, wave power and wind power acting on offshore structures are considered to have a strong correlation.[16)] These instances require generation of random numbers considering the correlation between the basic variables generated. For such methods, refer to References.

Random fields such as these need to be generated for reliability evaluations concerning inhomogeneous continuums modeled by random fields, such as ground, in particular. For an overview of typical methods and methods for reliability evaluations using easy modeling, Honjo, Otake, Kato (2012), Otake, Honjo (2012), and their references are useful.[17), 18)]

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